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National Institute of Technology Karnataka, Surathkal
Odd Semester, 2013 - 2014
MA939 Functional Analysis
Problem Sheet - 9

Date : 11.09.2013 Last Date of Submission : 16.12.2013

Answer **ALL** questions.

1. **Necessary and sufficient conditions for continuous.** Let X and Y be normed spaces and $T : X \to Y$ be a linear operator. Prove that the following are equivalent.
   (a) T is continuous at 0.
   (b) T is continuous at every $x \in X$.
   (c) T is uniformly continuous.
   (d) T is a bounded operator (there exists $M > 0$ such that $\|Tx\| \leq M\|x\|$ for all $x \in X$.)
   (e) T is a bounded function on $B[0, r]$ for some $r > 0$.
   (f) T sends null sequences in X to null sequences in Y.
   (g) T sends convergent sequences in X to convergent sequences in Y.
   (h) T sends Cauchy sequences in X to Cauchy sequences in Y.
   (i) T sends bounded sequences in X to bounded sequences in Y.
   (j) The null space of T is closed in X and the linear operator $\widetilde{T} : X/N(T) \to Y$ defined by $\widetilde{T}(x + N(T)) = Tx$, $x \in X$, is continuous.
2. **Sufficient conditions for continuous.** Let T be a linear operator from a normed space X into a normed space Y. Prove the following statements.
   (a) If X is Banach and the inverse image of the closed unit ball in Y is closed in X, then T is continuous at 0.
   (b) If the image of every null sequence is bounded, then the operator is continuous.
   (c) If $\sum_n Tx_n$ is a convergent series in Y whenever $\sum_n x_n$ is an absolutely convergent series in X, then T is continuous.
3. Let $T : (\mathbb{R}, \|.\|_1) \to (\mathbb{R}, \|.\|_\infty)$ be a map defined by $T(x, y) = (2x + 3y, x - y)$. Compute the operator norm of T, by optimization techniques.
4. Show that two complex Banach spaces can be isomorphic as real Banach spaces but they may fail to be isomorphic as complex Banach spaces.
5. Prove or disprove. c is isomorphic to c_0.
6. Prove or disprove. c is not isometrically isomorphic to c_0.