Introduction to Computational Mathematics

P. Sam Johnson

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Introduction

Numerical analysis is a process to develop and evaluate the methods for computing required numerical results from given numerical data.

The given data may be called **input** information, the results obtained may be called **output** information.

The methods applied to obtain output information may be termed as **algorithm**.

Objectives

In practical applications, one would finally obtain results in a numerical form. For example, from a set of tabulated data derived from an experiment, a system of linear algebraic equations is to be solved.

The aim of numerical analysis is to provide efficient methods for obtaining numerical answers to such problems.

We focus mainly on methods of numerical analysis for solving problems arising in the following different areas of higher mathematics.

Interpolation

Given a set of data values

$$(x_i, y_i), \quad i = 0, 1, 2, \ldots, n$$

of a function

$$y = f(x),$$

where the explicit nature of $f(x)$ is not known, it is often required to find the value of y for a given value of x, where $x_0 < x < x_n$.

This process is called **interpolation**.

If this process is carried out for functions of several variables, it is called **multivariate interpolation**.

Curve Fitting

Given a set of data values

$$(x_i, y_i), \quad i = 0, 1, 2, \ldots, n$$

a smooth curve can be drawn to pass through near the plotted points. Such a curve is called an **approximating curve**.

The equation of this curve may be taken as an approximate relation between x and y and it is called an **empirical equation**.

The general problem of finding equations of approximating curves which fit a given data is called **curve fitting**.

The method is to fit a curve which passes through the data points and then use the curve to predict the intermediate values. This problem is usually referred to as **data smoothing**.

Numerical Differentiation and Integration

It is often required to determine the numerical values of

- $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ at some x in $[x_0, x_n]$, and
- $\displaystyle\int_{x_0}^{x_n} y \, dx$

for the given set of data values

$$(x_i, y_i), \quad i = 0, 1, 2, \ldots, n.$$

The explicit nature of $y(x)$ is not known.

For example, if the data consist of the angle θ (in radians) of a rotating rod for values of time t (in seconds), then its angular velocity and angular acceleration at any time can be computed by numerical differentiation formulae.

Numerical Solution of Algebraic and Transcendental Equations

The equations of the form

$$f(x) = 0$$

are called **algebraic and transcendental** according as $f(x)$ is purely a polynomial in x or contains some other funtions such as exponential, logarithmic and trigonometric functions etc.

The problem of solving nonlinear equations of the type

$$f(x) = 0$$

is frequently encountered in engineering.

Numerical Solution of Ordinary Differential Equations

Engineering problems are often formulated in terms of an ordinary differential equation.

For example, the mathematical formulation of a falling body involves an ordinary differential equations. In most cases, exact solutions are not possible and numerical methods have to be adapted.

Numerical Solution of Systems of Linear Equations

Simultaneous linear algebraic equations occur in various fields of science and engineering. Cramer's rule is very much useful for smaller systems but the method is found to be impracticable for large systems, since the calculations are tedious. To solve such equations, there are numerical methods, which are particularly suited for computer operations.

The problem of solving systems of linear algebraic equations and the determination of eigenvalues and eigenvectors of matrices are major problems of disciplines such as differential equations, fluid mechanics, theory of structures, etc.

References

1. **Richard L. Burden and J. Douglas Faires**, *Numerical Analysis - Theory and Applications*, Cengage Learning, Singapore.
2. **Kendall E. Atkinson**, *An Introduction to Numerical Analysis*, Wiley India.
3. **David Kincaid and Ward Cheney**, *Numerical Analysis - Mathematics of Scientific Computing*, American Mathematical Society, Providence, Rhode Island.
4. **S.S. Sastry**, *Introductory Methods of Numerical Analysis*, Fourth Edition, Prentice-Hall, India.